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SAT/PSAT Study Materials

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Examples from the Algebra Section

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Substitution and Simplification of Algebraic Expressions

You will be required to know the meanings of many terms in order to work the problems in the algebra section. We will begin our study with a review of some of the basic terms.

What is an algebraic expression?

A **numeric expression** represents the relationship of various numbers.

$$3(5-7)^2+2$$

An **algebraic expression** represents the relationship between numbers and unknown quantities (variables).

$$3(5-7)^2+y$$

What is the difference between an **expression** and an **equation**?

An equation links two expressions with an equal sign.

So how do the writers of the PSAT / SAT test your knowledge of algebraic expressions?

Some problems will ask you to substitute a value into an expression to find the answer. The expression may be given to you in symbols, or it could be written in words that you have to change into symbols. You will then need to correctly simplify the resulting numeric expression. Other problems will give you a complex expression and ask you to simplify it without substituting a value in for the variable.

The most important concept when simplifying either type of problem is to follow the order of operations. You probably know the famous mnemonic:

Please **E**xcuse **m**y **d**ear **A**unt **S**ally.

Parentheses

Exponents

Multiplication and **D**ivision from left to right

Addition and **S**ubtraction from left to right.

Some students forget that the word **Parentheses** represents all symbols of grouping. Any symbol that groups numbers together or separates one group from another must be dealt with before working with exponents or other operations. Parentheses (), brackets [], and braces { } do not cause problems for most students. Students (and some teachers) overlook the fraction bar as an important symbol of grouping. The fraction bar separates the fraction into two groups of numbers and operations: the numerator and the denominator.

Example #1

If $x = 7$, what is the value of $3x - 15$?

The answer is found by substituting 7 in place of the x and then simplifying.

$$3(7) - 15 = 21 - 15 = 6$$

Correct answer: **6**

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	0	0	0	
1	1	1	1	
2	2	2	2	
3	3	3	3	
4	4	4	4	
5	5	5	5	
6	6	6	6	
7	7	7	7	
8	8	8	8	
9	9	9	9	

Example #2

If $z = \frac{1}{2}$, what is the product of $3z$ and $4z$?

Start by translating the words into symbols and then substituting $\frac{1}{2}$ for z

$$\begin{aligned}(3z)(4z) &= \left(3 \cdot \frac{1}{2}\right)\left(4 \cdot \frac{1}{2}\right) \\ &= \left(\frac{3}{2}\right)(2) \\ &= 3\end{aligned}$$

Correct answer: **3**

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		<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
	0	0	0	
1	1	1	1	
2	2	2	2	
3	3	3	3	
4	4	4	4	
5	5	5	5	
6	6	6	6	
7	7	7	7	
8	8	8	8	
9	9	9	9	

Example #3

$3 + x - (2x - 3)$ is equivalent to which of the following expressions?

- A) $-x$ B) $x + 6$ C) $6 - x$ D) 6 E) $x - 6$

$$\begin{aligned}3 + x - (2x - 3) &= 3 + x - 2x + 3 \\ &= -1x + 6 \text{ or } 6 - x\end{aligned}$$

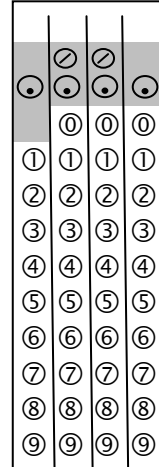
Correct answer: Choice C

Example #4

If $d = 3$ what is the value of $(d + 5) - \left(\frac{d - 3}{d + 3}\right)^2$

$$\begin{aligned}
 (3 + 5) - \left(\frac{3 - 3}{3 + 3}\right)^2 &= 8 + \left(\frac{0}{6}\right)^2 \\
 &= 8 + 0^2 \\
 &= 8
 \end{aligned}$$

Correct answer: 8



Substitution Extension #1:

If the question is asking for the value of a specific variable, then one option is to plug in each of the answer choices and determine which actually works. When using this method, it is wise to begin with a letter other than choice A. If the answer choices are in increasing order, then beginning with choice C will give you the chance to eliminate two extra choices. Based on whether the value needs to be larger or smaller, eliminate either choices A & B or D & E.

Example #1

If $3x - 5 = x + 3$, what is x ?

- A) 1 B) 2 C) 3 D) 4 E) 5

Plugging in $x = 3$ results in $9 - 5 \neq 6$, so choice C is not the answer.
 The left hand side is too small, so let's try a larger value of x .
 Plugging in $x = 4$ results in $12 - 5 = 4 + 3$, so choice D is the correct answer.

Example #2

At the ballpark, a hot dog costs \$1.00 more than a large drink. If the price for 6 hot dogs and 4 large drinks is \$36, what is the cost of a hot dog?

- A) \$1.00 B) \$2.00 C) \$3.00 D) \$4.00 E) \$5.0

Beginning with choice C, the price of a hot dog is \$3, and the price of a large drink would be \$2.

$6 \cdot \$3 + 4 \cdot \$2 = \$26$ Choice C is not large enough.

If the price of a hot dog is \$4, then the price of a large drink would be \$3.

$6 \cdot \$4 + 4 \cdot \$3 = \$36$ Choice D is the correct answer.

Substitution Extension #2:

If the question is instead asking for you to perform an operation on one or more variables, then plugging in the answer choices is not going to work. Instead, you get to pick values that meet any requirements set forth in the problem in order to find the answer.

Example #1

If $x + 5 = y$, then $x - 3$ is how much less than y ?

- A) 0 B) 2 C) 4 D) 6 E) 8

If we decide to make $x = 5$, then $y = 10$ and $x - 3 = 2$. Now the question is, “2 is how much less than 10?”
The answer is (E) 8.

Example #2

If $\frac{-x}{y} = 1$, then $x + y =$

- A) 0 B) 1 C) x D) y E) $2x$

If we choose x to be equal to 2, then y must equal -2 to satisfy the condition.
Then we plug our values into the question: $x + y = 2 + -2 = 0$. The correct answer is choice A.

Example #3

If x , y and z are all positive integers and $x < y < z$, which of the following must have the greatest value?

- A) $\frac{z}{x+y}$ B) $\frac{x}{y+z}$ C) $\frac{y}{x+z}$ D) $\frac{x+y}{z}$ E) $\frac{y+z}{x}$

Let's choose three integers that satisfy the conditions. For example:

If you choose $x = 1$, $y = 2$, and $z = 3$.

Choice (A) is equal to 1

Choice (B) is equal to $\frac{1}{5}$

Choice (C) is equal to $\frac{1}{2}$

Choice (D) is equal to 1

Choice (E) is equal to 5

The correct answer is choice E.

Substitution Extension #3: Double Substitution

Some of the more difficult problems you will encounter have variables in all of the answer choices. These questions are meant to test whether you understand the problem solving process. Most students find it difficult to solve algebraic questions that have nothing but variables in them. To help make these problems easier, we can substitute numbers of our choosing for each variable. The problem has changed from an abstract problem to a concrete problem. Depending on your abilities, there are two options as to how to proceed with the problem.

Example #1

The accountant for a large company is shopping for a paper shredder. One of the shredders he is considering can shred p pages in m minutes. If he purchased this shredder, how many pages could he shred in 7 hours?

- A) $\frac{7p}{m}$ B) $\frac{420p}{m}$ C) $\frac{420m}{p}$ D) $\frac{7p}{60m}$ E) $\frac{60p}{7m}$

Option 1: (takes less time but more thought)

1. Work the problem with the variable values you have chosen (do not simplify).
2. Substitute the variables into the work and then simplify if necessary.

Step 1: If we choose $p = 5$ and $m = 2$ then the shredder can shred $\frac{5 \text{ pages}}{2 \text{ minutes}} = 2.5$ pages per minute

The question asks about pages per hour, so we multiply by $\frac{60 \text{ minutes}}{1 \text{ hour}}$ and then multiply by 7 hours. The process would look like this:

$$\frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{7 \text{ hours}}{1} = \frac{5 \cdot 60 \cdot 7}{2} \text{ pages}$$

In the first step you are working the “new” problem based on the values you chose for the variables. The reminder not to simplify is to help you focus on the process more than just the answer.

Step 2: Substitute the variables back into the process before simplifying and you get:

$$\frac{5 \cdot 60 \cdot 7}{2} \text{ pages} = \frac{p \cdot 60 \cdot 7}{m} \text{ or } \frac{420p}{m}$$

The correct answer is choice B.

In the second step the values in the “answer,” which you have not simplified, change back to variables. Then by simplifying this expression, you should have one of the answer choices. These questions ask for the formula to find the answer to a generic question. By working a sample problem with numerical values you discover the answer to one specific question. In addition to the answer to the specific problem, you have also discovered the process of how to get to the answer. Then by changing the values back into variables, we arrive at the generic process or formula that was requested.

Option 2 (takes more time but is easier for most students)

1. Find the answer to the problem with numbers substituted for each variable.
2. Determine which of the answer choices will give you the same answer by plugging your variable values into all of the answer choices.

Step 1: Step 1 uses the same process as in Option 1

If we choose $p = 5$ and $m = 2$ then the shredder can shred

$$\frac{5 \text{ pages}}{2 \text{ minutes}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{7 \text{ hours}}{1} = \frac{5 \cdot 60 \cdot 7}{2} = 1050 \text{ pages}$$

Step 2: Substitute $p = 5$ and $m = 2$ into each of the answer choices.

- A) $\frac{35}{2}$ B) 1050 C) 168 D) $\frac{7}{24}$ E) $\frac{150}{7}$

Only one of the 5 answer choices equals the value of 1050 pages from step 1. Answer choice B is the correct answer.

You have to plug the chosen values into all five choices because there is a slight chance that more than one of the answer choices may result in the same value. If more than one choice “works” the first time, try a different set of values in just the choices that worked the first time.

Three rules help to minimize the chances of getting more than one answer choice that works.

- **Avoid values that are in the problem or answer choices.**
- **Avoid choosing 1 or 0 for any of your variables.**
- **Avoid choosing the same value for two or more variables.**

Functions

What is a function?

This is a simple question, yet the answer is not quite as simple. For our purposes, a function is a relationship between two variables in which for each possible value of the independent variable there is a single value of the dependent variable. Some students will say that it is an equation written with an $f(x)$ instead of y . While it is true that $f(x)$ and y are interchangeable in a function, this is not the most critical feature of a function. Plus, there are some equations where you cannot substitute $f(x)$ for y . So let's try to understand our working definition and try to answer some other important questions about functions.

Independent vs. Dependent Variable

One key to understanding functions is familiarity with some of the vocabulary associated with functions. First for discussion is the relationship between the independent variable and the dependent variable. One of the variables will react to changes in the other value. The variable that reacts is called the dependent variable, and the variable that causes the other to change is the independent variable. Typically we assign $f(x)$ or y the job of being the **dependent variable**, and we let x perform the role of the **independent variable**.

Domain vs. Range

The **domain** of a function lists all of the possible values of the independent variable. The **range** of a function lists all of the possible values of the dependent variable. This leads many students to associate domain with x values and range with y values. In a standard math class domain = x and range = y will be sufficient to answer most questions, but when dealing with the devious writers of the SAT / PSAT tests, this association may lead to an incorrect answer.

Predictable Values of the Dependent Variable

An important characteristic of a function is that the values of the dependent variable are predictable. Every time the independent variable is a specific number, the dependent variable will become one specific value. In other words, for each value of the independent variable (each value in the domain), there is **one** value of the dependent variable (one value in the range). If we are working with $y = f(x)$, for each possible x value, there is one value of y . There will never be two values in the range that are associated with a single value in the domain. In other words, there will never be two different values of y associated with a single value of x in any function of x .

Some students may be thinking, “I think he just said the same thing five different ways!” Some might be thinking, “I think you might be right, but I still don’t get it.” If you are a member of the first group, skip the rest of this paragraph. If you are still confused, stick around and we’ll see if we can’t clarify the situation. Let’s look at some examples of what I mean by the previous paragraph. If $f(3) = 7$, then $f(3)$ will always equal 7. It will not become equal to 8 at a later time. $f(3)$ is a constant value that will not change unless the definition of the function changes. $f(4)$ cannot be equal to 0 once and 2 the next time I try to use the definition. However, it is possible that both $f(2)$ and $f(3)$ are equal to 7. A real world example would be the amount of sales tax you pay on the purchase of a taxable item. The final price $f(x)$ can be found by multiplying x by 1.08. If you had to purchase an item worth \$2 before taxes, you will always pay \$2.16 including taxes. That means that $f(2) = \$2.16$, and it will not change unless the tax rate changes.

What is an example of an equation that would not be a function?

Let’s look at an example of an equation where a single independent value is paired with two different dependent values. If x is the independent variable and y is the dependent variable for the equation $x^2 + y^2 = 10$, then when $x = 1$ there are two possible values of y , 3 and -3. A real world example for a relationship that is not a function is the entrance fee for an amusement park. The price to buy four tickets is not always the same. Discount coupons, age of the four people, time entering the park, and many other factors could influence the price. Therefore the price of the entrance fee is not a function of the number of tickets you are buying.

Why do we often see a function written with $f(x) =$ instead of $y =$?

The notation using $f(x)$ clearly shows the relationship between a specific value of the domain and a specific value of the range. For example, $f(3) = 7$ tells me that when the independent value (x) is 3, then the dependent value (y) is 7. In other words $f(3) = 7$ corresponds to the point (3, 7) on the graph of $f(x)$.

When trying to find the value of a function for specific values of the independent variable, you will need to substitute the value into the definition of the function. For example, if $f(x) = x^2 - 3$, then $f(7)$ could be found by substituting a 7 everywhere there is an x in the definition. If we do this substitution, we find that $f(7) = (7)^2 - 3$ or $f(7) = 46$.

As quickly as possible use the definition of a function, $f(x) = x^2 - x$, and make the initial substitution for each of the values.

$$f(x) = x^2 - x$$

$$f(1) = \underline{\hspace{10em}}$$

$$f(2) = \underline{\hspace{10em}}$$

$$f(-1) = \underline{\hspace{10em}}$$

$$f(z) = \underline{\hspace{10em}}$$

$$f(z+1) = \underline{\hspace{10em}}$$

$$f(\text{blue}) = \underline{\hspace{10em}}$$

$$f(\text{I Love Math!}) = \underline{\hspace{10em}}$$

Some people will claim that it was a dirty trick to include the last example. Yes, it was included so that at some point in your life you will write “I Love Math!” It was also included to make a very specific point: It does not matter what the value is inside the parenthesis; you will follow the same process. Hopefully you wrote:

$$(\text{I Love Math!})^2 - (\text{I Love Math!})$$

Is $x = y^2$ a function?

Your Algebra or Pre-Calculus teacher may have told you that this equation is one example of an equation that is not a function. That statement is not 100% accurate. $x = y^2$ is not a function if x is the independent variable. However, if you consider y to be the independent variable, then it **is** a function and it could be written in function notation as $x = f(y) = y^2$. If I were reading that out loud, I would say, “**X** is the function of **y** that is equal to y^2 .” Be Careful!!

What do the various numbers in a quadratic function mean?

There are two basic ways that a quadratic function can be written:

$$f(x) = ax^2 + bx + c \quad \text{and} \quad f(x) = a(x-h)^2 + k$$

Each of the values **a**, **b**, **c**, **h**, and **k** serves a specific role in the formula. In fact there are times when it will be easier to work with one or the other of the basic formulas; so, you must know them both.

If you are concerned with where the graph of the function intersects the axes, the first of the formulas is more useful. If you need to know how to shift the graph of $y = x^2$ up/down and left/right to get the function, the second formula is more helpful. Both functions can be used to find the direction the parabola opens, the highest point of the graph, the lowest point of the graph, and the vertex of the parabola.

The formula $f(x) = ax^2 + bx + c$ can help us find the intercepts of the graph without much effort.

The graph of the parabola will cross the y axis when $y = c$. The x intercepts can be found by

factoring, the quadratic formula $\left(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$ or by graphing (if you are using a graphing

calculator). If a is positive, then the parabola will open up, but if a is negative, the parabola will open down. To help remember the direction the graph opens, use a to classify the graph as positive or negative. Positive people (and parabolas) smile ☺ while negative people (and parabolas) frown ☹.

The formula $f(x) = a(x-h)^2 + k$ helps us see how to change $y = x^2$ into the parabola we are working with. I suggest you apply the changes according to the order of operations. The h tells us about the shift right or left. $(x-2)^2$ is shifted right 2 while $(x+2)^2$ is shifted left 2. The a will determine the direction the parabola will open just as it does in the other formula. Also, we will need to multiply each y value from the original graph by a in order to get an intermediate y value for the new graph. I call it an intermediate value because we then need to add k to get the final y value.

The vertex of the parabola can be found from either of the formulas with relative ease. The vertex of

the first form occurs at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$ while the vertex of the second form is at (h, k) . Obviously

it is easier to find the vertex from the second formula, but either format can be used. The SAT / PSAT has yet to ask specifically for the vertex of a parabola, so why are we spending valuable time discussing it? The ability to find the vertex makes it easy to answer questions about the greatest or least possible value of a quadratic function. The vertex of the parabola will be the highest point or the lowest point on the graph depending on whether the graph opens down or up. The SAT/ PSAT writers have asked about the maximum or minimum value. If the parabola opens downward, the maximum occurs at the vertex and no minimum value exists. If the parabola opens upward, the graph hits its minimum value at the vertex and never reaches a maximum value.

The table at the top of the next page attempts to summarize the information that we have already discussed. Some of the information is useful only when discussing quadratic functions, but some of the ideas can be applied to help with any function. The concept that we will see used the most with other functions is the idea of shifting an original curve to make it into a different curve.

Important Feature	$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$
Direction the parabola opens	Opens upward if a is positive Opens downward if a is negative	
Line of symmetry	$x = \frac{-b}{2a}$	$x = h$
Vertex	$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$	(h, k)
Y intercept	$(0, c)$	$f(0)$
Shifts		Right h (Left if $+h$) Up k (Down if $-k$)

Other functions can be shifted left and right by adding or subtracting values inside the function or shifted up and down by adding values outside the function. For example, if I wanted to move the graph of $f(x)$ 3 units to the right, I would write the new function as $f(x-3)$. If it needed to move 3 units left, I would use the function $f(x+3)$. To move $f(x)$ up three, write the expression as $f(x)+3$. To move it down 3, the expression would be $f(x)-3$.

Now for some guided practice!

Example #1

If $f(x) = x^2 - 4x$ what is the value of $f(3)$?

- A) -3 B) -1 C) 0 D) 1 E) 3

The question asks for the value of $f(3)$ when given a definition in terms of $f(x)$. We need to change all of the x 's in the definition into 3's. This results in the statement that $f(3) = 3^2 - 4(3)$. When we simplify the right side of the equation we find that $f(3) = 9 - 12 = -3$.

Choice A is the correct answer

Example #2

If $f(x) = x^2 - 4x$ what is the value of $f(y+1)$?

- A) $(y-1)^2$ B) $y^2 + 2y$ C) $y^2 - 2y + 2$ D) $y^2 - 4y - 3$ E) $y^2 - 2y - 3$

We need to start this problem in the same way we began the previous problem by substituting a $y + 1$ for each x : $f(y + 1) = (y + 1)^2 - 4(y + 1)$. Now we need to simplify the right hand side of the equation. Make sure you review the factoring problems in the quadratic section, because we will now make use of one of the formulas you learned.

We substituted $y + 1$ for each x in the definition:

$$\begin{aligned} f(y + 1) &= (y + 1)^2 - 4(y + 1) \\ &= y^2 + 2y + 1 - 4(y + 1) \\ &= y^2 + 2y + 1 - 4y - 4 \\ &= y^2 - 2y - 3 \end{aligned}$$

Expand $(y + 1)^2$ using $(x + y)^2 = x^2 + 2xy + y^2$

Distribute the -4 to the y and the $+1$.

Combine like terms to arrive at the answer.

Notice the answer could have been factored as $(y - 3)(y + 1)$. Remember that currently you should not expect to factor a problem like this, but in the near future it may become necessary.

Choice E is the correct answer.

Example #3

According to the following table of values, what is the value of $f(2)$?

x	1	2	3	4
$f(x)$	2	7	1	8

- A) 1 B) 2 C) 3 D) 7 E) 8

If we want to find $f(2)$, we notice that the x in $f(x)$ has been replaced with a 2. Therefore we realize that $x = 2$. When $x = 2$, $f(x)$ is 7.

Choice D is the correct answer.

Example #4

Use the table of values from example #3 to help answer the following question. If

$g(x) = 2 \cdot f(x) - 1$, what is the value of $g(3)$?

- A) 1 B) 2 C) 5 D) 7 E) 8

First change all of the x 's in the function definition to 3's.

$$g(3) = 2 \cdot f(3) - 1$$

Then replace $f(3)$ with the value from the table.

$$g(3) = 2(1) - 1$$

Simplify the right hand side of the equation.

$$g(3) = 1$$

Choice A is the correct answer.

Example # 5

Use the table of values from example #3 to help answer the following question. If

$g(x) = 2 \cdot f(x) - 1$ and $g(k) = 3$, what is the value of k ?

- A) 1 B) 2 C) 3 D) 4 E) 5

The first step is to substitute a k for each x .

$$g(k) = 2 \cdot f(k) - 1$$

Then set the right side of the equation equal to 3.

$$3 = 2 \cdot f(k) - 1$$

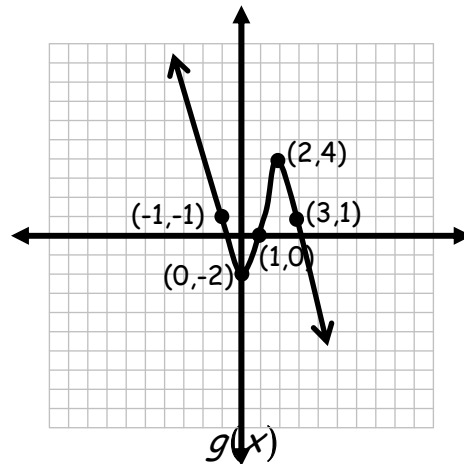
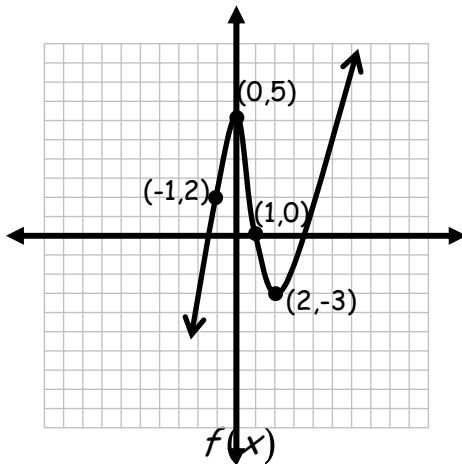
Next, solve for $f(k)$ by adding 1 and dividing by 2.

$$1 = f(k)$$

Look at the table to determine what x value is paired with a 2.

Choice A is the correct answer.

Use the following graphs to answer examples 6 – 9.



Example #6

What is the value of $f(2)$?

- A) -3 B) -1 C) 1 D) 3 E) 4

The trick to solving this question lies in remembering that $f(2)$ represents the y value of a point when the x value is 2. By looking at the graph of $f(x)$, we see that the point is $(2, -3)$. Therefore the correct answer is choice A, -3.

Example #7

What would be the value of $f(f(1))$?

- A) -2 B) 0 C) 1 D) 2 E) 5

This is a fun little two step problem where we need to work from the inside out. First we need to find $f(1)$, the y value when x is 1. We notice that $f(1) = 0$, so we substitute zero for $f(1)$ in the original problem to find that the question now asks for the value of $f(0)$. Our final answer is that y is equal to 5 when x is zero.

Choice E is the correct answer.

Example #8

What is the value of $g(f(1))$?

- A) -2 B) 0 C) 1 D) 2 E) 5

This problem is similar to Example #7 except that it involves both of the graphs. We begin by finding the value of $f(1)$. We substitute zero into $g(f(1))$ since $f(1) = 0$. Now we need to find the value of $g(0)$, the y value of graph g when x is zero. $g(0) = -2$.

Choice A is the correct answer.

Example #9

What is the value of $f(g(1))$?

- A) -2 B) 0 C) 1 D) 2 E) 5

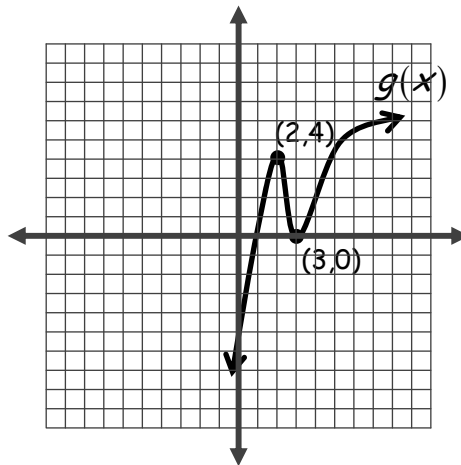
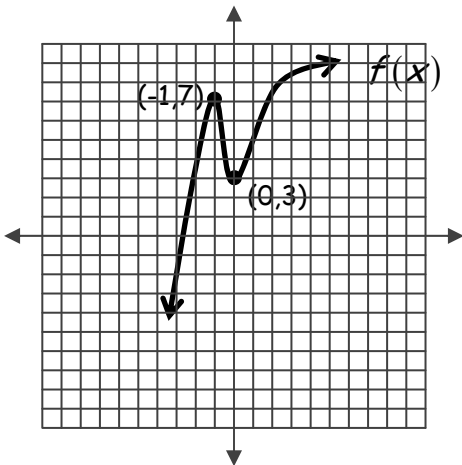
Once again we work with both graphs, but this time we start with graph g since $g(1)$ is the inner most piece. Y is 0 when x is 1 on graph g so $g(1)=0$. Now moving to graph f we need to find the value of $f(0)$. From example 7, we remember that $f(0)=5$.

Choice E is the correct answer.

Use the graphs below to answer example #10.

Example #10

If $g(x) = f(x+h)+k$, what is the value of $h+k$?



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②	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
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Since the graph of $g(x)$ results from shifting $f(x)$ right 3 and down 3, you may want to conclude that the answer is negative 9. Remember that the grid will not allow a negative answer. Also notice that the formula ($g(x) = f(x+h)+k$) is not the normal transformational formula we looked at earlier ($f(x) = a(x-h)^2 + k$). Remember that in our transformational formula if h is positive (you are subtracting), then the graph moves to the right, and if k is positive the graph moves up. Since the signs are different in this problem, the directions of movement will be changed. To move right 3, h must be a -3. To move down three, k must be -3. Thus h times k will be 9.

9 is the correct answer.

Example #11

The price that an employee at an electronics store will pay for an item can be described as a function of the retail price. The function $E(P)$ is the price an employee will pay after their discount if P is the retail price. If $E(P) = 0.80P$, what would be the employee's price for a \$100 camera?

- A) \$20 B) \$40 C) \$60 D) \$80 E) \$100

Neophytes would like to select choice E since the question stated that it is a \$100 camera. Now that we have eliminated choice E as being too obvious, let's use what we know about functions to answer this word problem. \$100 must be the retail price so we can put it in the place of P in our function definition. $E(\$100) = 0.80(\$100)$ which is \$80, choice D.

Practice Problems

1. If $f(a) = a^3 - a^2 + a - 1$, what is the value of $f(-3)$?

- A) -40 B) -17 C) -10 D) 17 E) 40

2. If $g(k) = \frac{k^3 + k}{k + 3}$, what is the value of $g(2)$?

- A) $\frac{8}{5}$ B) $\frac{10}{3}$ C) 5 D) 2 E) $\frac{16}{5}$

3. If $K(z) = \frac{(z+1)^2}{(z-1)^2}$, what is the value of $K(-1)$?

- A) -2 B) 0 C) $\frac{1}{2}$ D) 2 E) does not exist

Geometry

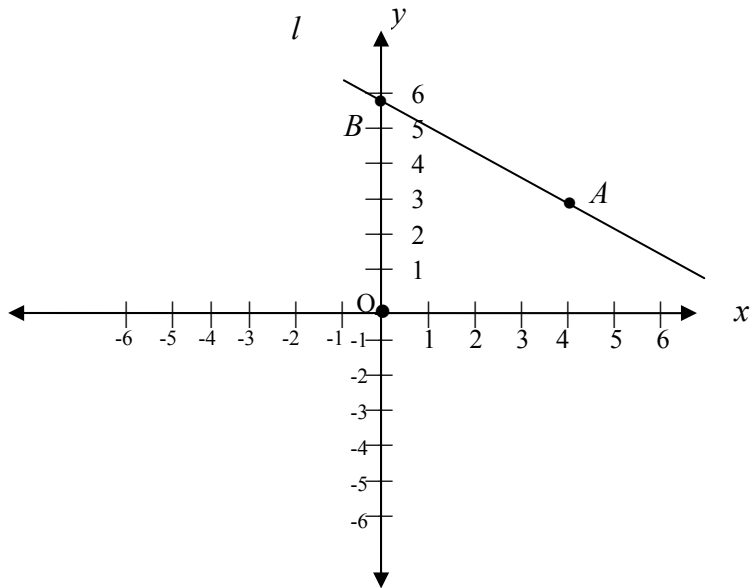
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RETURN TO MAIN MATH MENU

Geometric Notation

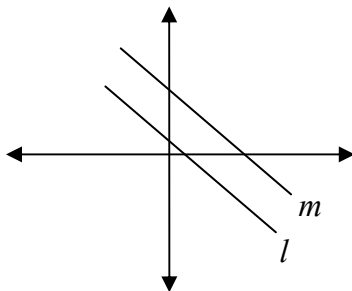
Several questions in the geometry section will require you to recognize and use geometric notation for points and lines, line segments, rays, angles and their measures, and lengths. You should be familiar with the standard coordinate plane and how figures are graphed on it. Look closely at the figure below.



Notice that the x and y axes meet at point O , also called the **origin**. Every point on the coordinate plane has a location described by **coordinates** in the form (x, y) . These coordinates are also called an **ordered pair** because they contain a pair of values in a specific order. In the figure above, point A is located at $(4, 3)$, and point B is located at $(0, 6)$. The origin is always located at $(0, 0)$. There is a line drawn connecting points A and B , line l . We could also identify the line by a different notation, \overline{AB} . Quite often, the SAT test writers will name lines by lower case letters such as l or m .

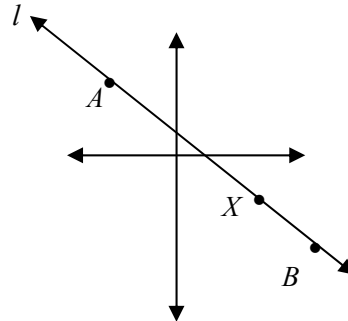
Note: Lines will be named either by a lower case letter (line l) **OR** by two points on the line (\overline{AB}), but never both at the same time.

When two lines are parallel, i.e. they never intersect, you will see the symbol “ \parallel ” between their names to show this. For example, if lines l and m are parallel, you will see $l \parallel m$.

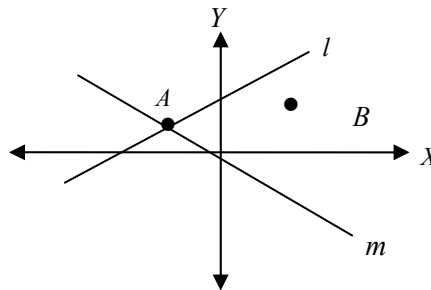


Practice Problems

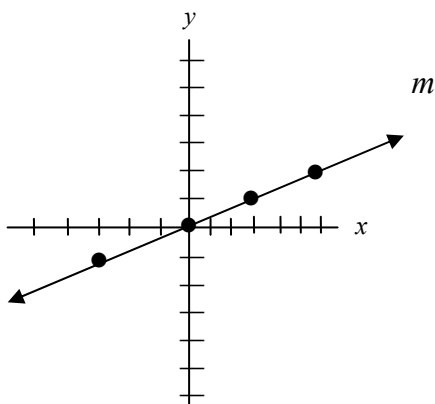
1. A line is drawn through three points as shown. Which of the following is **not** an accurate name for the line?
 A) \overline{AX} B) \overline{BX} C) \overline{AL} D) \overline{AB} E) l



2. Which of the following statements is valid for the figure shown?
 A) $l \parallel m$
 B) $m \parallel l$
 C) Line l contains point B.
 D) Line m contains point A.
 E) Line m contains point B.



3.



Which of the points listed below is not on line m shown at left?

- A) $(-3, -1)$
 B) $(0, 0)$
 C) $(2, 6)$
 D) $(3, 1)$
 E) $(6, 2)$

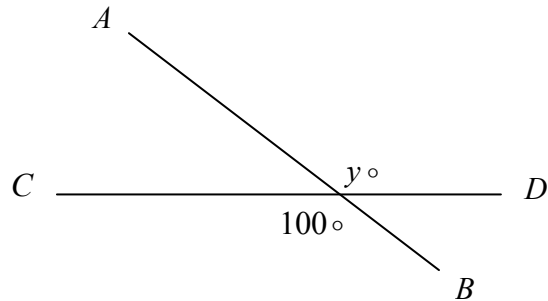
Angles in the Plane

Many questions you will find on the test will ask about the angles that are formed in a plane by intersecting lines, line segments, and rays. There are two kinds of angles that you must be familiar with before you can solve any of the problems.

Vertical Angles

Vertical angles are two opposite angles formed by two intersecting lines.

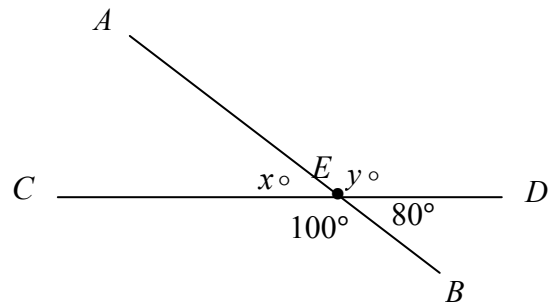
Line \overline{AB} intersects line \overline{CD} , forming four separate angles. The angles opposite each other are called **vertical angles**. Vertical angles always have the same measure. In the diagram above, $y = 100$.



Supplementary Angles

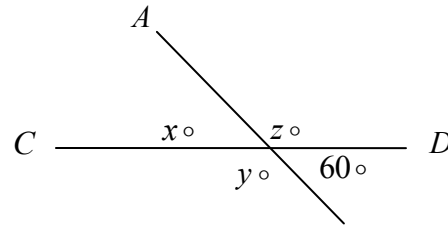
Supplementary angles are two angles that form a straight line when they share a common side. The sum of any two supplementary angles equals 180° . Notice that angle $\angle AEC$ and angle $\angle AED$ form a straight line. Therefore, $x + y = 180$. In the diagram, there are four pairs of supplementary angles:

- $\angle AEC$ and $\angle AED$
- $\angle AED$ and $\angle DEB$
- $\angle DEB$ and $\angle BEC$
- $\angle BEC$ and $\angle CEA$



Test Your Understanding

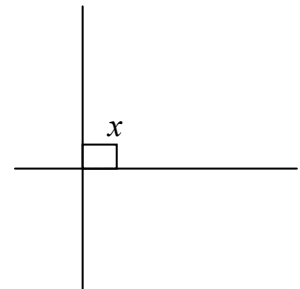
1. What is the value of z ?
2. What is the value of x ?
3. What is the value of $x + z$?
4. What is the value of $x + z - y$?



Answers: 1. 120 2. 60 3. 180 4. 60

Right Angles

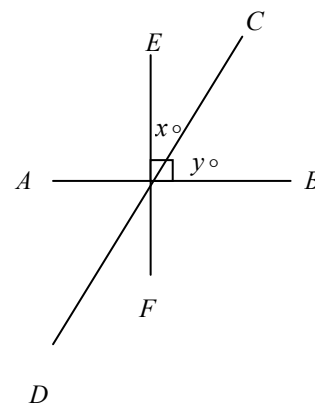
A right angle is formed when two lines intersect and form an angle of 90° . In such a case, the lines are called **perpendicular lines**. Each of the four angles formed at such an intersection is a right angle, and each is equal to 90° . In the diagram shown, a square at the point of intersection indicates a right angle. In the diagram you can see that $x = 90$.



When two lines are perpendicular, we use the symbol " \perp ". For example, in the diagram below, $\overline{EF} \perp \overline{AB}$.

Complementary Angles

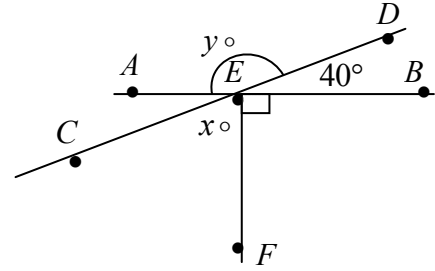
Two angles whose sum equals 90° are called **complementary angles**. In the diagram, line \overline{CD} intersects line \overline{AB} creating angles, shown by x and y . Because the two angles formed have a sum of 90° (since together they form a right angle), the two angles x and y are called **complementary angles**.



Example #1

In the figure \overline{AB} and \overline{CD} intersect at point E, and $\overline{AB} \perp \overline{EF}$. What is the value of $y + x$?

- A) 100 B) 150 C) 180
D) 190 E) 270

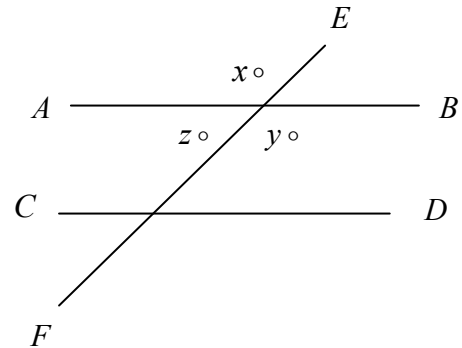


You can see from the diagram that $y = 140^\circ$ since $\angle AED$ is supplementary to $\angle DEB$. You can find the measure of $\angle AEC$ by using vertical angles. $\angle AEC$ has a measure of 40° , so $\angle CEF$ must be $90^\circ - 40^\circ = 50^\circ$. You should also see from the diagram that $x + 90 + 40 = 180$. In either case, $y + x = 140 + 50 = 190$, which is Choice D.

Parallel Lines

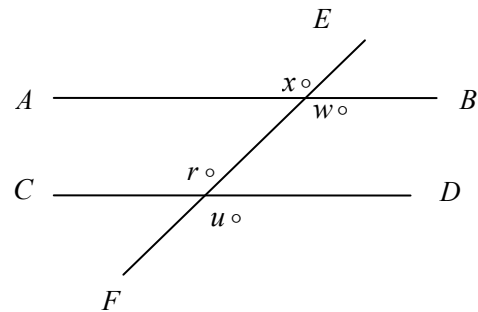
Other types of angles are created when a line intersects a pair of parallel lines. Eight separate angles are created by this intersecting line, also called a transversal.

Each angle is related to the others in different ways. You already know that x and y are equal because they are vertical angles. You also know that $x + z = 180$ because they are supplementary angles. The same is true of $z + y$.



Corresponding Angles

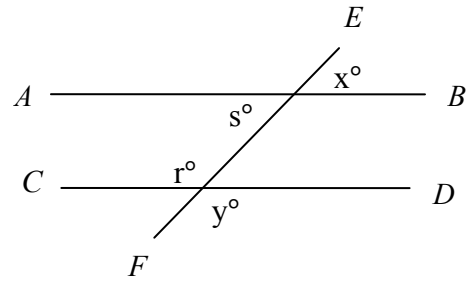
Corresponding angles get their name because they correspond to each other in relation to their placement on the parallel lines. For example, x is the angle to the left of the line that intersects line \overline{AB} and r is the angle to the left of the line that intersects line \overline{CD} . Angles r and x are **corresponding angles**. Corresponding angles always have the same measure. If $x = 100$, then $r = 100$. Two other corresponding angles on the diagram are w and u . Because they are corresponding angles, $w = u$.



Same Side Interior or Same Side Exterior Angles

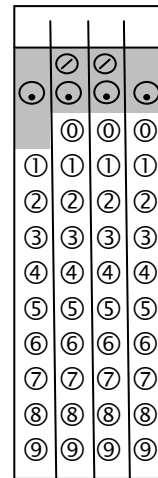
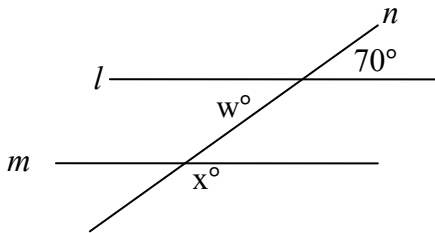
There are other supplementary angles that are created when a line intersects a pair of parallel lines. The sum of the interior angles s and r is 180° . Angles s and r are called **same side interior** angles.

The sum of angles x and y also equals 180° . Angles x and y are called **same side exterior** angles. You must memorize these facts.



Example #2

In the diagram, lines l and m are parallel and are intersected by line n . What is the value of $x - w$?



We can see that $w = 70$ because the angles are vertical angles.

To find the value of x , notice that the angle marked x° and the angle marked 70° are on the same side of n , but outside of lines l and m . These are same side exterior angles, so they must be supplementary (the sum of their measures adds up to 180°). Thus, x must be $180 - 70 = 110$. Then $x - w = 110 - 70 = 40$.

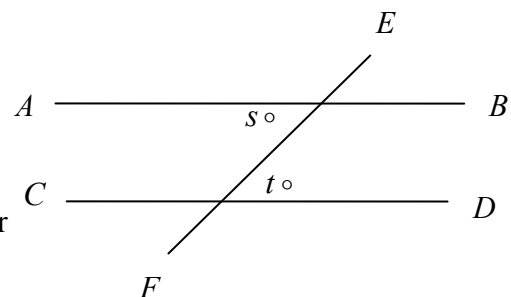
Alternate Interior Angles

You saw the interior angles r and s in the diagram above.

Many questions you will see on the test deal with **alternate interior** angles. These are very important.

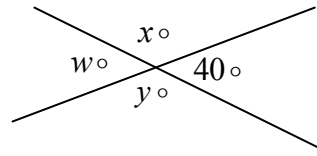
There are two things to remember in identifying the angles. First, they have to be interior angles. Second, they have to be on opposite sides of the intersecting line.

Angles s and t are alternate interior angles. Alternate interior angles are always equal in measure ($s = t$).

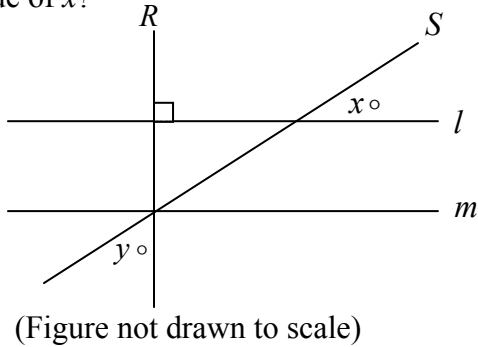


Practice Problems

1. For the two intersecting lines shown, which of the following must be true?
 I. $w + x = y + 40$ II. $x < y$ III. $2w = 80$
- A) I only B) II only C) I and III D) II and III E) I, II, III

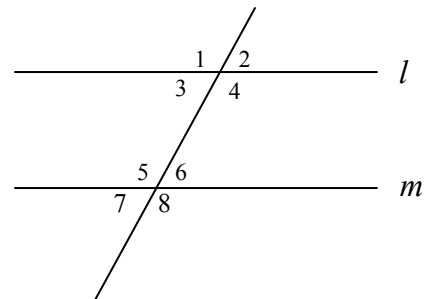


2. In the figure shown, $l \parallel m$ and $l \perp R$. If $y = 2x$, what is the value of x ?

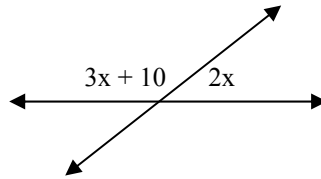


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⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

3. In the figure, if $l \parallel m$, then the measure of angle 2 is equal to the measure of which other angle?
- A) 1 B) 4 C) 5 D) 7 E) 8

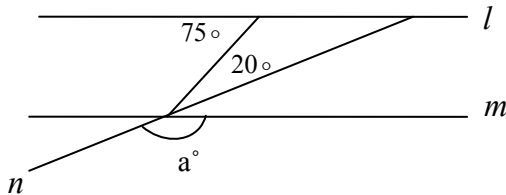


4. For the supplementary angles shown, what is the value of x ?



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④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

5. Lines l and m are parallel. What is the value of a ?

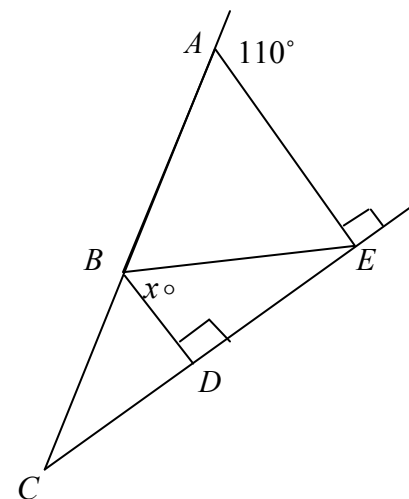


- A) 160 B) 125 C) 115
D) 105 E) 95

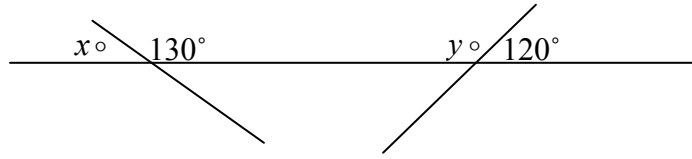
6. In the figure $\overline{BD} \perp \overline{CE}$, $\overline{AE} \perp \overline{CE}$, and $AE = EB$. What is the value of x ?

- A) 20 B) 40 C) 55 D) 70 E) 85

(Figure not drawn to scale)

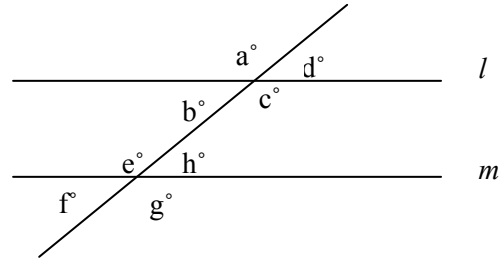


7. What is the value of $x + y$?
 A) 10 B) 90 C) 110 D) 200 E) 250



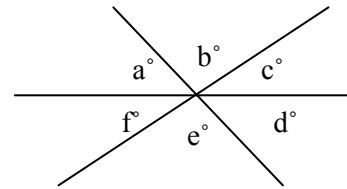
8. In the figure, $l \parallel m$, and $h = 80^\circ$.
 What is the value of $a + c + f + g$?

- A) 200 B) 280 C) 300
 D) 360 E) 380

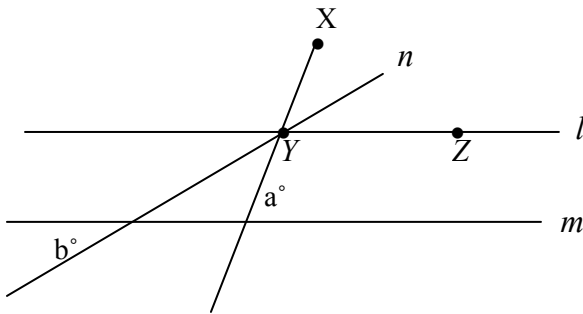


9. If $a + b + d = 180^\circ$, then which of the following statements is **not** true?

- A) $a + d + e = 180^\circ$ B) $2a + b = 180^\circ$ C) $c = b$
 D) $c = d$ E) $a = c$



10. In the figure $l \parallel m$, and n bisects $\angle XYZ$.
 If $30 < b < 40$, what is one possible value for a ?



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3 – 4 – 5 Triangles and other Triples

Some common values that are used as sides to a right triangle are called **Pythagorean triples**. These are used often on the PSAT/SAT. You should memorize the four most common triples:

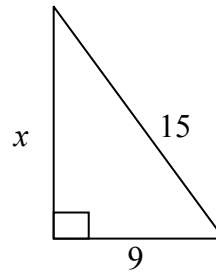
- 3-4-5
- 5-12-13
- 8-15-17
- 7-24-25

Pythagorean Triples

If you multiply one of these triples by a constant, you get another triple. For example, multiplying 3-4-5 by 4 gives 12-16-20. There are an infinite number of Pythagorean triples, but the four to the left are the most important.

Example #1

From the given figure at the right, find the value of x .



Note that both 9 and 15 are divisible by 3.

Thus, $\frac{9}{3} = 3$ and $\frac{15}{3} = 5$. What triple do you know

that has one leg of length 3 and a hypotenuse of 5?

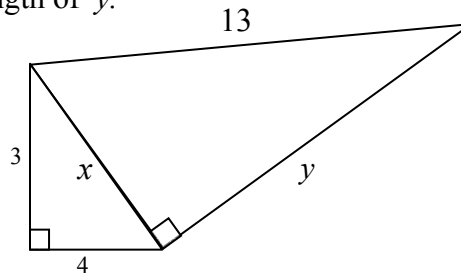
3-4-5! Multiplying by 3 gives us a new triple:

9-12-15. Therefore, $x = 12$.

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④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

Example #2

From the figure at the right, determine the length of y .



The smaller triangle is a 3 – 4 – 5 triangle, so $x = 5$.

That gives the larger triangle a leg of 5 and a hypotenuse of 13. This is the 5 – 12 – 13 triple. Therefore, $y = 12$.

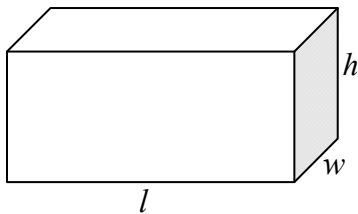
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⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

Solid Geometry

Solid geometry covers the area of three-dimensional geometric figures. When we study solids in geometry, we sometimes split them into 2 categories: solids with 2 bases, and solids with 1 base.

Solids with 2 bases

A solid with 2 bases is called a *prism*. A simple box is a great example of a rectangular prism, because all 6 sides are rectangles. To find the volume (V) of a rectangular prism, we multiply the length \cdot width \cdot height.



* Rectangular prisms are also called right prisms because the adjoining edges meet at right angles.

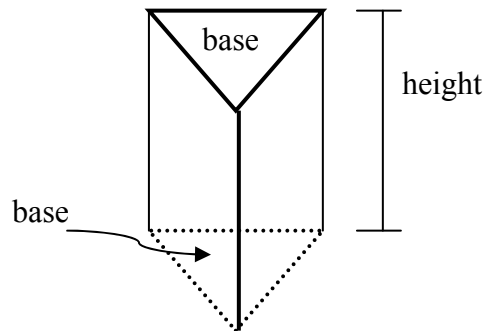
$$V = l \cdot w \cdot h$$

The most common solid used on the PSAT/SAT test is the cube. A cube is a type of rectangular prism in which all edges are equal. Since all edges are equal, the formula for the volume of a cube is $V = s \cdot s \cdot s$, or

$$V = s^3$$

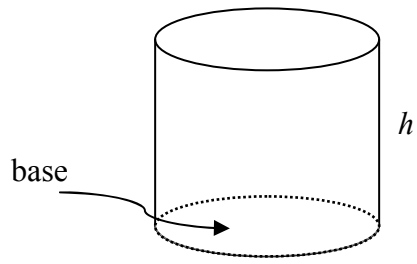
A prism is named for the shape of its base. For example, the following figure is called a *right triangular prism*.

As you can see, the base of the figure is a triangle. The height of any right prism is the length of the edge joining the 2 bases.



A **cylinder** is a solid with 2 equal bases in the shape of a circle. The formula for the volume of a cylinder is $V = \text{area of base} \cdot \text{height (h)}$. Since the base is a circle, this formula can also be written $V = \pi r^2 \cdot \text{height}$, or

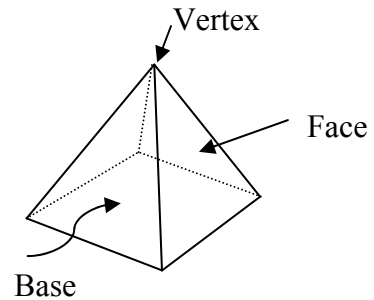
$$V = \pi r^2 h$$



Solids with 1 base

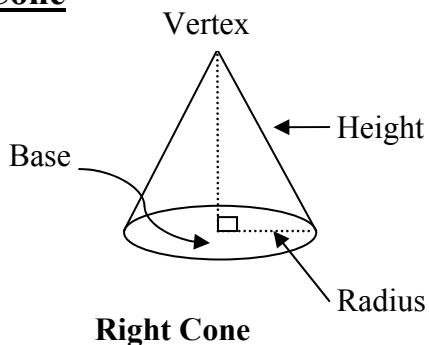
Pyramid

A pyramid is a solid with one base in the shape of a polygon. The faces of a pyramid are triangles that meet at one point called the vertex. If the base is a regular polygon, then the triangular faces are all congruent, and the solid is called a regular pyramid. Pyramids are also named by the shape of their bases.



Regular Square Pyramid

Cone



Right Cone

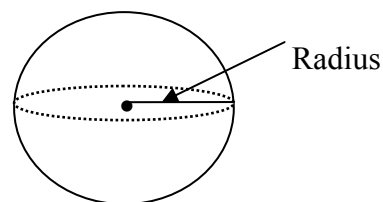
A cone is similar to a pyramid, except that the base is a circle, instead of a polygon. If the height of the cone is perpendicular to the base, we have a **Right Cone**.

The formula for the volume of a cone or a pyramid is $V = \frac{1}{3} \text{Base} \cdot \text{height}$, or $V = \frac{1}{3} B h$

$$V = \frac{1}{3} B h$$

Sphere

Another type of solid has no faces and is called a **sphere**. A basketball is an ideal example of a sphere. All radii are equal in a sphere. To find the volume of a sphere, we use the formula $V = \frac{4}{3} \pi r^3$.

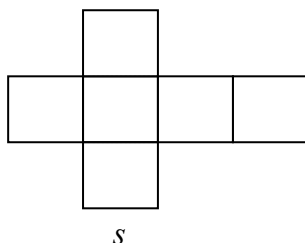
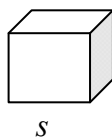


Surface Area

The surface area of a solid is the sum of the areas of all its faces. A good formula to memorize is the surface area of a cube. A cube has 6 congruent faces. Since each face is a square, the area of one face is s^2 . Since the cube has 6 equal faces, the surface area of a cube is $6s^2$.

$$\text{Surface area of a cube} = 6s^2$$

Sometimes you will see the surface area of an object represented by cutting the solid along an edge, opening it up, and laying it flat. This is called a **net** of the solid. Study the figure of the cube below and the net drawn to the side of it.



Surface area may be easier to understand if you have a net of the solid to view.

Example #1

The volume of a cube is 27 cubic in. What is its surface area?

- A) 3 in^2 B) 9 in^2 C) 27 in^2 D) 54 in^2 E) 108 in^2

$$\begin{aligned} \text{Volume} &= s^3 \\ 27 &= s^3 \\ 3 &= s \end{aligned}$$

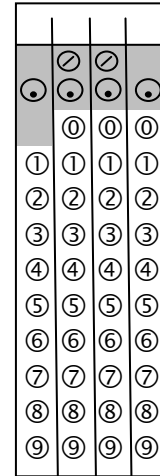
$$\begin{aligned} \text{Surface Area (SA)} &= 6s^2 \\ \text{SA} &= 6(3)^2 \\ \text{SA} &= 6 \cdot 9 \\ \text{SA} &= 54 \text{ sq. in. (choice D)} \end{aligned}$$

Example #2

What is the volume in cubic feet of a cube that has a surface area of 24 square ft.?

$$\begin{array}{l} \text{Surface Area (SA)} = 6s^2 \\ 24 = 6s^2 \\ 4 = s^2 \\ 2 = s \end{array} \qquad \begin{array}{l} \text{Volume} = s^3 \\ V = 2^3 \\ V = 8 \text{ cu. ft.} \end{array}$$

* Note that the volume is measured in cubic units while surface area is measured in square units.

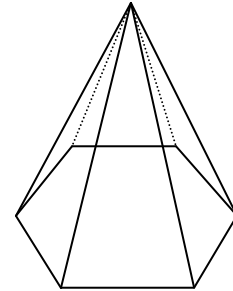


Example #3

How many faces does a regular hexagonal pyramid have?

- A) 4 B) 5 C) 6 D) 7 E) 12

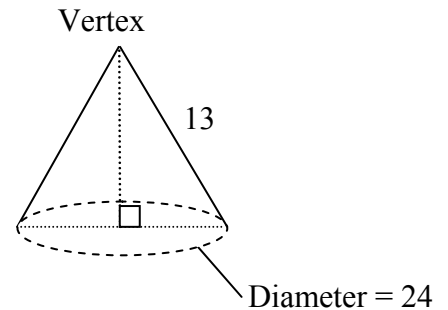
Since the base of the pyramid is a hexagon, the figure will have 6 other triangular faces.
6 triangular faces + 1 base = 7 total faces, choice D.



Example #4

A right cone has a diameter of 24. The distance from the edge of the base to the vertex is 13. What is the height of the cone?

- A) 5 B) 6 C) 7 D) 10 E) 15



The radius of the cone, along with its height, form a right triangle with the distance from the base edge to the vertex. Since the diameter = 24, the radius = 12.

Using the Pythagorean Theorem or the triples, h = 5, choice A.

Example #5

A cube has an edge of $2x$. A larger cube has an edge of $4x$. What is the ratio of the volume of the larger cube to the volume of the smaller cube?

- A) 1:2 B) 1:8 C) 2:1 D) 4:1 E) 8:1

$$\begin{aligned} V_{\text{smaller cube}} &= (2x)^3 = 8x^3 \\ V_{\text{larger cube}} &= (4x)^3 = 64x^3 \end{aligned} \qquad = \frac{64x^3}{8x^3} = \frac{8}{1}$$

Choice E is the answer.

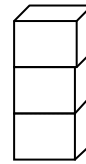
Practice Problems

1. What is the length of the edge of a cube whose surface area is $96e^2$?

- A) $16e$ B) $8e$ C) $8e^2$ D) $4e^2$ E) $4e$
-

2. Three cubes, all with edges 2 cm., are stacked on top of the other as shown. What is the surface area of the resulting solid?

- A) 40 B) 56 C) 48
D) 28 E) 96

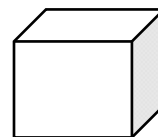


3. A right prism has bases in the shape of octagons. What is the total number of faces of the prism?

- A) 16 B) 12 C) 10 D) 9 E) 8
-

4. The volume of a cube is 64cm^3 . What is the total length of all the edges of the cube, in centimeters?

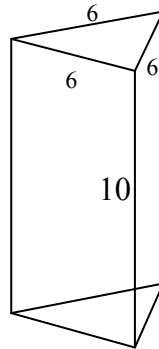
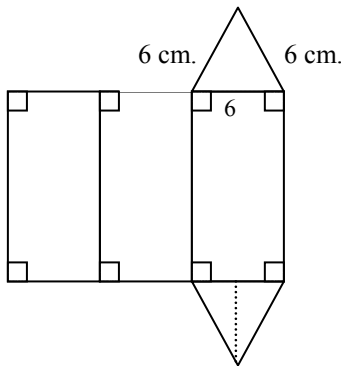
- A) 48 B) 32 C) 64
D) 40 E) 96



5. Two right circular cylinders have equal heights. The radius of the base of the first cylinder is 3, while the radius of the base of the second cylinder is 4. What is the ratio of the volume of the smaller cylinder to the volume of the larger cylinder?

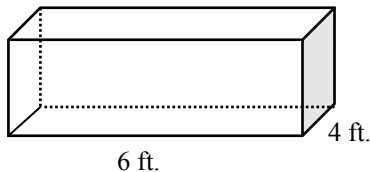
- A) $\frac{3}{4}$ B) $\frac{4}{3}$ C) $\frac{9}{16}$ D) $\frac{16}{9}$ E) $\frac{1}{2}$

6. The figure shown is a net of a box in the shape of a triangular prism. The triangular bases are equilateral triangles with a perimeter of 18 cm. The height of the prism is 10 cm. What is the area of **one** rectangular face, in square cm.?



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0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

7. A large fish tank has a rectangular base that is 6 ft. by 4 ft. The tank currently contains water at a height of $2\frac{1}{2}$ ft. What volume of water (in cubic ft.) must be added to the tank to make the water level measure 3 ft. deep?



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3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9